Filtrations, canonical formulas, and axiomatizations of superintuitionistic and modal logics

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There are two main tools for establishing the finite model property for modal and superintuitionistic logics: the methods of standard and selective filtrations. For superintutionistic logics standard filtration algebraically corresponds to taking the implication-free reduct of Heyting algebras and selective filtration corresponds to the join-free reduct. The key property that makes these methods work is that these reducts are locally finite. These finite model property proofs can be turned into axiomatization methods using canonical formulas. These formulas were introduced model-theoretically by Zakharyaschev building on the work of Jankov, de Jongh, Fine and Rautenberg. Zakharyaschev's canonical formulas algebraically correspond to join-free reducts of Heyting algebras. Every superintuitionistic logic is axiomatizable by these formulas. Important subclasses of canonical formulas are Jankov-de Jongh, subframe and cofinal subframe formulas giving rise to classes of join-splitting, subframe and cofinal subframe logics, respectively. In this talk, I will also discuss recently introduced stable canonical formulas, which algebraically correspond to implication-free reducts of Heyting algebras. Every superintuionistic logic is also axiomatizable by stable canonical formulas. These formulas give rise to a new class of stable logics.

Modal logic counterpart of Zakharyaschev's canonical formulas axiomatizes all extensions of $\mathsf{K4}$ (this was later generalized to all extensions of the weak transitive logic $\mathsf{wK4}$). This technique is based on the method of selective filtration for transitive modal logics. While selective filtration is very effective in the transitive case, it is less effective for K. This is one of the reasons why canonical formulas do not work well for K. I will discuss how to generalize the technique of stable superintuitionistic canonical formulas to the modal setting. Since the technique of filtration works well for K, we show that this new technique is effective in the non-transitive case as well. However, due to the lack of the master modality in the case of K we need to work with rules as opposed to formulas. I will define stable canonical rules and show that each normal modal logic is axiomatizable by stable canonical rules. For normal extensions of $\mathsf{K4}$ we prove that stable canonical rules can be replaced by stable canonical formulas, thus providing an alternative to Zakharyaschev's axiomatization.